

MATHEMATICS D

Paper 4024/11

Paper 1

Key Messages

To do well on this paper, candidates should have covered the entire examination syllabus. Formulae must be remembered correctly and appropriate methods used to solve each question. It is important to read the questions on the paper carefully, highlighting the key points and the form in which the answer is required, and give the result to a suitable degree of accuracy. All relevant working for each question should be shown clearly and calculations performed accurately.

General Comments

The paper contained questions accessible to all candidates. Candidates generally did well on simultaneous equations, questions involving money and linear equations. Areas for improvement include upper and lower bound (particularly for subtraction), probability, proofs for similar triangles, non-calculator work involving angles greater than 90° , multistep problems such as **Question 17** and **Question 24**, and problems set in context.

Presentation of the work was usually good with most candidates showing sufficient clear working. Incorrect answers are best crossed out and replaced rather than written over. There is no need to delete working if it is not to be replaced as the original attempt may gain some credit. A check should be made to ensure that the answer in the working has been correctly transcribed to the answer space.

There was more of a problem this year with candidates writing their working in pencil, deleting it and leaving just the answer. It is important that candidates note the instruction on the cover page that 'the omission of essential working will result in loss of marks' as opportunities to score some marks for steps within the working will be lost. Erasing pencil working also leaves pieces of rubber on the script, which can make the script difficult to read, particularly the decimal point. Numbers should be written clearly, particularly 5, which can look like 3, and 1, which can look like 7.

There were some obtained answers which candidates should have realised must be wrong. In **Question 7a** probability cannot be more than 1 so an answer such as 1.1 cannot be correct. Answers involving numbers of people or prices cannot be negative, relevant in **Question 23b** and **Question 23c**. If given a particular value for π in the question, as in **Question 24c**, this should be used.

Comments on Specific Questions

Question 1

- (a) This part was answered well. Those who carried out the operations in the correct order were nearly always successful although some thought 3 was the answer to 6 divided by 3.
- (b) Candidates understood that a common denominator was required to carry out the subtraction and a high proportion reached the correct answer. A few then incorrectly tried to cancel $\frac{8}{45}$.

Answers: (a) 19 (b) $\frac{8}{45}$

Question 2

- (a) Recognition of the use of 'area of a triangle' frequently resulted in reaching the correct answer, although $\frac{1}{2}$ was sometimes missing from the formula. Common wrong answers were 5 cm (from assuming triangle BCD was isosceles) and 6 cm (halving 12).
- (b) Those who recognised that the perpendicular height of triangle ABD was the value found in (i) usually gave the correct or follow through answer. There were a few attempts to find DB by using Pythagoras' Theorem, often then abandoned or linked with $\frac{1}{2} ab \sin C$, and a number of candidates did not attempt this part.

Answers: (a) 8 (b) 48

Question 3

- (a) This part was well answered although 5^2 was sometimes evaluated as 10.
- (b) Candidates who attempted this question frequently misread the question and thought that N (rather than the lowest common multiple of N and $2^2 \times 5^2 \times 7$) was $2^2 \times 3 \times 5^2 \times 7^2$. This gave common wrong answers of 21 (from dividing $2^2 \times 3 \times 5^2 \times 7^2$ by $2^2 \times 5^2 \times 7$) or 14700 (the lowest common multiple of the two given numbers).

Answers: (a) 700 (b) 147

Question 4

- (a) This part was done well. Incorrect placement of the decimal point, arithmetic errors or a division were the usual errors.
- (b) This part was not quite as well done and had similar errors to (a).

Answers: (a) 320 (b) 150

Question 5

The correct answer was sometimes obtained despite rather confused working and poor expressions. Pythagoras was recognised by many candidates although there were a few attempts to use area of a triangle or trigonometry. Unnecessary attempts were made to find the value of $\sqrt{50}$. Common misunderstandings came from using PS as 25 or 12.5. Misuse of Pythagoras led to $\sqrt{84}$ from adding the two squares. A number of candidates did not attempt the question.

Answer: 4

Question 6

- (a) Common wrong answers were 307, 30600 and 31000.
- (b) The most successful candidates were those who answered the question by simply adjusting the decimal point using the information given in the question, rather than those who carried out a division.

Answers: (a) 30700 (b) 0.538

Question 7

Even though the question stated 'does **not** pass Physics', many candidates used 0.6, producing calculations such as 0.7×0.6 , $0.7 + 0.6$, $0.7 - 0.6$, $\frac{(0.7+0.6)}{2}$ or $\frac{7}{13}$. The value 0.4 was sometimes seen in the working but it was not always used or $0.7 + 0.4 = 1.1$ was given. It was not very common for 0.7×0.4 to be calculated correctly with errors such as $\frac{7}{10} \times \frac{4}{10}$ written as $\frac{28}{10}$ seen.

Answer: 0.28

Question 8

- (a) This question proved very difficult for many candidates with calculations such as $90 + 57$ or $90 - 57$ or an answer of -57 . The information given that $90 < y < 180$ was often ignored. A considerable number of candidates were unable to start the question.
- (b) This part was answered well. The common error was to subtract 2 from 33 once $5a - 2 = 33$ had been reached.

Answers: (a) 123 (b) 7

Question 9

- (a) Most responses were correct with $p = kq$ commonly used to find k and then r , but some gave the answer for k rather than r .
- (b) This part proved much more difficult with common wrong answers of x , \sqrt{x} , k/x or a number only.
- (c) This part was the most difficult with few realising that if L is doubled, M is 2^3 times larger. Incorrect answers were frequently 2 or 4 or $3 \times 2 = 6$.

Answers: (a) 11 (b) x^2 (c) 8

Question 10

- (a) This part was generally correct but common wrong answers were 2 and 8 or -2 and -8 .
- (b) The correct answer was fairly common as were the wrong answers of 3 or 17, generally due to arithmetic errors.
- (c) A number of answers were the solution of the inequality only, with no reference made to the given list.

Answers: (a) -8 and 2 (b) -3 (c) -2 , 0, 2

Question 11

- (a) Common incorrect answers for the lower bound included 0.79, $0.8 - 0.5 = 0.3$ and $0.8 - 0.1 = 0.7$.
- (b) Correct answers were very rare. The usual error was to find $6 - 0.8 = 5.2$ then subtract either 0.05 or 0.5. Others realised they needed the lower bound of the total mass so found $5.5 - 0.75 = 4.75$.

Answers: (a) 0.75 (b) 4.65

Question 12

- (a) Candidates frequently recognised that the first step was to find the total for the group of five numbers. Occasionally 3.8×5 was calculated as 190. Some attempted to create a list of 5 numbers whose mean was 3.8 but this was a less successful approach. Common errors were to find $3.8 + \frac{(3+6)}{2}$ or $(3.8 + 3 + 6)$ divided by 2, 3 or 7.
- (b) 4 was a common incorrect answer, often given without justification or found from n^{th} term $= \frac{(7+1)}{2} = 4$. A number of candidates felt they needed to find all 7 numbers in order to find the median rather than recognising that, since the original median was 3, the middle two numbers were both 3. A number of candidates were unable to attempt this part.

Answers: (a) 4 (b) 3

Question 13

- (a) The mode was well understood but some gave 15 as the answer.
- (b) Many candidates knew how to start the question but made arithmetic errors such as $0 \times 5 = 5$ or adding up the frequencies incorrectly (even though the total frequency was given in the question). The answer should be left as 2.08 as the mean does not have to be a whole number. Having reached $\frac{104}{50}$, answers were sometimes then given as 2.8 or 2.4. Others found $\frac{50}{6}$ or divided fx by 15 or 6.

Answers: (a) 3 (b) 2.08

Question 14

- (a) The correct answer was generally given by the better candidates, whereas others failed to deal with both the negative power and the square root of 9 with -3 , 3 and 81 common answers.
- (b) This part was usually correct, with 9999 the common wrong answer. Some thought 10^0 was 0 or found $10^{3-0} = 10^3 = 1000$.
- (c) The fractional power within an equation caused problems although some recognised that cube roots were involved. Answers of 2 and 8 were common.

Answers: (a) $\frac{1}{3}$ (b) 999 (c) 4

Question 15

The initial step, of squaring both sides, caused some problems, with 4, 8 or 2 given for the square of 4. Some candidates kept in 4^2 throughout their solution. The step of collecting x terms to one side and number terms to the other side of the equation was more successful and there were many correct answers.

Answer: $\frac{17}{16d - c}$

Question 16

- (a) The answer was generally correct. The common error was to use the power 5. Candidates should ensure they use 7.53, not 7.5.
- (b) Many candidates did not recognise the need to write the two masses with the same powers of 10 and simply added the numbers and added the powers.

Answers: (a) 7.53×10^{-5} (b) 6.045×10^{24}

Question 17

The more popular approach was to replace x by $3 - t$ in the given equation. Problems were then experienced in expanding $(3 - t)^2$, often giving $9 - t^2$ or $9 + t^2$. Those who tried to solve $(3 - t)^2 = 4$ often gave $3 - t = 2$ only.

Answer: 1 and 5

Question 18

- (a) Well answered.
- (b) Although the formula for pattern p was sometimes found, the formula for pattern n was more common and some answers were left unsimplified. The common wrong answer was $p + 5$, with others giving $p + 6$ or $6p + 5$ or a numerical answer.

Answers: (a) 21 (b) $5p + 1$

Question 19

- (a) Incorrect answers of $180^\circ + 65^\circ = 245^\circ$ and $180^\circ - 115^\circ = 65^\circ$ were common. The question did not require the angle to be measured.
- (b) Although there were some correct answers, most candidates did not realise that perpendicular bisectors of AB and BC were required. Common errors were to draw medians or a circle around B or to join and shade triangle ABC . The most common region to be shaded when the correct perpendicular bisectors were drawn was only within triangle ABC and not to the edge of the land region.

Answers: (a) 295° (b) Perpendicular bisectors of AB and BC with region around B shaded.

Question 20

- (a) (i) Those who realised that constant acceleration is equal to speed divided by time were generally successful. Common wrong answers were $\frac{4}{5}$ or $4 \text{ (m/s}^2\text{)}$.
- (ii) This part was less well done. Some gave an answer of 20, forgetting to add on their answer from (i). Others attempted to use the same formula as in (i) for the whole of 15 seconds giving $2 \times 15 = 30$ or gave $\frac{2}{15}$.
- (b) Those using the area of a trapezium were usually successful although some candidates misread the question and found the distance travelled for the whole journey. Others simply multiplied their answer to (aii) by 10 or left the answer blank.

Answers: (a)(i) 20 (ii) 40 (b) 300

Question 21

- (a) The pie chart was generally drawn accurately and labelled but often there was no working for the angles shown. A few drew an angle of 68° instead of the 72° they had calculated.
- (b) Many correct answers were seen. Some approached the problem by the longer method of first calculating the total cost and selling prices, rather than using $\frac{72 - 60}{60} \times 100$. A common error was to divide the profit by the selling price instead of by the cost price. Others stopped when they reached 120%. A number of candidates, who showed a correct method, thought $72 - 60 = 8$.

Answers: (a) Pie chart completed accurately and labelled with Bananas and Oranges (b) 20%

Question 22

- (a) In general, candidates successfully expanded the brackets but like terms were not always collected and there were some sign errors and answers such as $4 - 16x$, $16 - 9x^2$. A few candidates tried to solve an equation.
- (b) Although some candidates could factorise either the numerator or denominator, only a few correctly factorised both and then cancelled the common factor $(3x + 1)$ to give the correct answer. Most attempts involved cancelling $3x^2$ with $9x^2$ or cancelling all the x 's leaving just number terms.

Answers: (a) $16 - 9x$ (b) $\frac{x+5}{3x-1}$

Question 23

- (a) Most candidates were able to show the required answer but their presentation was often difficult to follow. A few tried to substitute 15 and 12 into $5a + 4c = 108$ and others made no attempt.
- (b) There were many correct answers. Those candidates who used elimination rather than substitution were generally more successful. Given the context of the question, candidates who obtained fractional or negative answers should have realised that their answers could not have been correct. Care should be taken to ensure that if c is found first, the answers are placed correctly in the answer space.
- (c) Candidates were not always able to relate this part to the answers found in (b). Those who did were usually successful although there were many candidates who did not attempt this part.

Answers: (a) $15a + 12c = 324$ seen (b) $a = 16$, $c = 7$ (c) 99

Question 24

- (a) Many candidates were able to use the isosceles triangle PQR to find angle PRQ as 31° but were unable to complete the multi-step calculation to find angle PTS and gave an answer of 68° . Others found $360 - (118 + 99)$ or assumed that angle $PTS =$ angle PQR or angle QRS . Some attempted to use the interior angles of a pentagon.
- (b) This part proved to be too difficult for nearly all candidates. Most failed to see the four identical triangles linked to O in order to obtain the 75° angles at the centre. A common error was to assume angle EDC was 30° (half of 60°) leading to ECD being 75° . As EDC did not look like 30° , others decided to double the 60° to 120° for EDC giving an answer of 30° . Some used the interior angle sum of a pentagon and divided by 5 to give EDC as 108° with a final answer of 36° . A less common error was to find the reflex angle at the centre (300°) then halving it to give EDC as 150° , resulting in a final answer of 15° .
- (c) Some candidates started with the correct formula but the common error was to use πr^2 rather than $2\pi r$. Some used the whole circumference rather than $\frac{1}{6}$ of it. Others made no attempt to answer the question.

Answers: (a) 112° (b) 37.5° (c) 12.56

Question 25

A number of candidates made little or no attempt at the parts of this question.

- (a) Few candidates were able to show any appropriate working. There was confusion between similarity and congruency. Many gave reasons related to the sides, rather than the angles. Some were able to identify the correct pairs of angles but not give relevant reasons.
- (b) There were some correct methods seen with sometimes a correct answer of 7.5. This should not be corrected to 8 in the answer space. Many simply found $2 + 5 = 7$.

- (c) Very few correct answers, or attempts using the correct method but using x^2 instead of x , were seen. Candidates did not recognise that, as ABE , DFE and CFB were similar then the ratio of their areas was equal to the ratio of the squares of their sides i.e. $4^2 : 2^2 : 6^2$. This result gives the ratio $4 : 1 : 9$ which could be used with x to give the answer.

Answers: (a) Two corresponding pairs of angles equated, with reasons (b) 7.5 (c) $12x$

MATHEMATICS D

Paper 4024/12

Paper 1

General comments

In order to do well in this paper, candidates need to

- have covered the whole syllabus;
- remember necessary formulae and facts;
- recognise, and carry out correctly, the appropriate mathematical procedures for a given situation;
- perform calculations accurately;
- shown clearly all necessary working in the appropriate place.

It was noticeable that there were many scripts where candidates had not attempted many questions and had made poor attempts at the others. Some candidates did not appear to bring geometrical instruments into the examination.

At times, candidates appeared not to read the questions carefully. They should be encouraged to pay careful attention to the wording, the numbers given, the units used and the units required in the answer.

Questions that proved particularly difficult were 9**(b)**, 12**(b)**, 19, 22**(c)(ii)**, 23**(c)**, 24**(d)**, 24**(e)**, 25**(c)(ii)**, and 26**(b)(ii)**.

It was noticeable that a significant number of candidates need to improve their ability to approximate and to use appropriate degrees of accuracy; and also to understand the integer class of number. Some candidates were very competent at performing standard techniques, yet seemed unable to recognise the appropriate mathematical procedure required for a given situation.

Candidates need to improve their skills in manipulating fractions and in basic arithmetic, particularly when negative numbers are involved. Work, that used correct methods, was sometimes spoilt by wrong calculations.

A noticeable error occurred when an expression is obtained, followed by some simplifying. An equals symbol is then introduced and an equation solved. For example:

$$\begin{aligned}75 + 15k - 150 \\15k - 75 \\15k - 75 = 0 \\15k = 75 \\k = 5\end{aligned}$$

Presentation of work was usually good. Candidates should bear in mind that it is to their advantage to make sure they provide sufficient working and that this working is set out neatly and legibly. This makes it possible for marks, where they are available, to be awarded for correct methods and intermediate results.

A few candidates did not heed the instructions on the front page - to write in dark blue or black pen. Except for diagrams and graphs, candidates must not write in pencil. Nor must they overwrite pencil answers in ink, as this makes a double image which can be difficult to read. Some candidates wrote in pencil and even erased their workings, often producing rubber and paper debris that interfered with the clarity of other answers, particularly with a decimal point and a negative sign.

Candidates should be made aware that only their final answer to each question should be written in the answer space. Alternative offerings and working should not be written there. When an answer is to be changed, it is far better to delete and replace the original one rather than attempt to write over it.

Care must always be taken to ensure that answers obtained in the working are accurately transferred to the answer space.

Comments on Specific Questions

Question 1

- (a) Usually answered correctly. Wrong answers were usually 0.09; 0.9; 90; 9.
- (b) Attempts at this question showed that some candidates need to improve their understanding of the order of operations.

The usual wrong answers were 4.8, from 3×1.6 ; 4.6, from $3 + 1.6$; -3.8 , from $5 - 6 - 2.8$.

Answer: (a) 0.009 (b) 1.8

Question 2

- (a) Candidates usually attempted this question correctly.

Errors were quite frequently made in the arithmetic involved. A few thought that all the items were sold for \$10, instead of for \$10 *each*.

- (b) Most candidates started with the expression $\frac{450 \times 5 \times 4}{100}$, or its equivalent, but not all were able to evaluate it correctly.

Answer: (a) 59.30 (b) 90

Question 3

Most candidates dealt competently with this question.

A common error was to evaluate $6\sqrt{4}$ as 24.

A few did not read the question carefully, and assumed that y varies *inversely* as the square root of x ; or directly as the square of x ; or directly as x .

Answer: 12

Question 4

- (a) The majority of candidates knew to substitute $-\frac{2}{5}$ for x .

Errors were sometimes made in evaluating the expression obtained.

A common wrong answer was 0.6, either from $1 + 4\left(-\frac{2}{5}\right) = 1 - \frac{8}{20} = \frac{12}{20}$, or from $\frac{5-8}{5}$.

Occasionally $1 + 4\left(-\frac{2}{5}\right) = 5 \times \left(-\frac{2}{5}\right) = -2$ was seen.

- (b) Many candidates seemed to know how to deal with this type of question, though some did not seem to understand the meaning of $f^{-1}(x)$.

A common wrong answer was $\frac{x+1}{4}$.

Some looked to find $f^{-1}\left(-\frac{2}{5}\right)$ or similar, resulting in a numerical value only.

Occasionally, an answer was given that was not expressed in terms of x .

Answer: (a) -0.6 (b) $\frac{x-1}{4}$

Question 5

- (a) Many correct answers were seen. Common wrong answers were 0.050500; 0.0504; 0.05; 0.050; 505.

- (b) Candidates who followed the instruction 'By writing each number correct to 1 significant figure' were usually successful, though some found it difficult to evaluate $\frac{1.8}{30}$ correctly.

Answer: (a) 0.0505 (b) 0.06

Question 6

Answers were very varied, with many candidates making an error with one or more of the elements. One of the most common errors was to obtain $\begin{pmatrix} 0 & 9 \\ -9 & 3 \end{pmatrix} - \begin{pmatrix} -2 & -10 \\ 8 & 2 \end{pmatrix}$ as a first step.

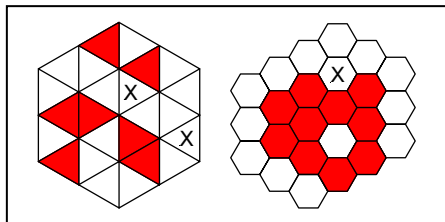
Answer: $\begin{pmatrix} -2 & -1 \\ -1 & 5 \end{pmatrix}$

Question 7

- (a) There were some correct answers. However, most attempts showed that many candidates need to acquire a better understanding of rotational symmetry. A common wrong answer was one that gave a figure with *line* symmetry.

- (b) This part was answered quite successfully.

Answers:



Question 8

Many candidates of all abilities answered this question correctly. Some arranged most letters in the correct order. A few confused '<' with '>' and gave the answer c, e, b, a, d .

Answer: d, a, b, e, c

Question 9

Responses indicated that many candidates need to gain a better understanding of bounds.

- (a) This part was well answered, many candidates appreciating that the lower bound is 5 m less than 60 m. The most common wrong answer was 59.5, candidates assuming Dave's throw to be correct to the nearest metre and not as specified in the question. Sometimes Dave's throw was taken correct to the nearest 10 cm, resulting in a response of 59.95. There were also occasional offerings of 59.995, 59.9 and 50.
- (b) There was much less success in this part, many failing to realise that the greatest difference resulted from Ed's longest throw and Dave's shortest. As a consequence, $(60 + 5) - (61 - 0.5) = 4.5$ or $65 - 61.5 = 3.5$ were quite common answers. Various misunderstandings which gave rise to incorrect differences included $60.5 - 59.5 = 1$; $60.5 - 55 = 5.5$; $61.5 - 60.5 = 1$.

Answer: (a) 55 (b) 6.5

Question 10

Generally answered well by those who know what is meant by standard form. Some candidates need to learn that standard form means $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

- (a) Common wrong answers were 45×10^{-7} ; 4.5×10^6 ; 45×10^7 ; 45×10^5 .
- (b)(i) Many candidates obtained 24×10^{15} in their working. Some either gave this as their answer, or else converted it to 2.4×10^{14} . Other wrong answers were 2.4×10^8 ; 2.4^{16} .
- (ii) Most candidates chose either to convert to an ordinary number, or change one number to the index of the other. The usual wrong answers were 2×10^1 , from $6 - 4$ and $8 - 7$; 1.5×10 , from $6 \div 4$ and $8 - 7$; 2×10^{15} from $6 - 4$ and $8 + 7$. Responses such as 56×10^7 or 5.6×10^7 were sometimes seen.

Answer: (a) 4.5×10^{-6} (b)(i) 2.4×10^{16} (b)(ii) 5.6×10^8

Question 11

Candidates who knew the laws of indices answered this question well. Many showed that they need to improve their knowledge of these laws.

- (a) This part was usually answered correctly. The usual wrong answers were 0; $\frac{3}{2}$.
- (b) Many candidates saw the index -1 as implying the reciprocal and gave the correct answer. The usual wrong answers were $-1\frac{1}{2}$; $\frac{3}{2}$; 1.5; $\frac{1}{1.5}$.
- (c) Many did square the 9 and square the x^3 to arrive at the correct answer. The usual errors were to give the constant as 9 or 18; to give the power of x as 3 or 5.

Answer: (a) 1 (b) $\frac{2}{3}$ (c) $81x^6$

Question 12

- (a) Most candidates seemed to know how to find the prime factors of a number. Some gave the correct answer. Others made errors in a division, or omitted to write down every factor they had found, or did not express their product as prime numbers and gave an answer such as $2 \times 9 \times 11$.
- (b)(i) Some candidates realised that the most direct method of finding the highest number that divides into both numbers is to study the prime factor form of the numbers. Others started by finding the values of M and N . Many did not seem to know what was required. Common wrong answers were 12600 (the LCM); 151200 ($= MN$); 25.
- (ii) Many did not seem to know that a cube number requires indices that are 3, or a multiple of 3. Successful candidates used the prime factor form and $2^2 \times 3 \times 5^2 \times k = x^3$ to observe that $k = 2 \times 3^2 \times 5$. A wide variety of wrong answers were seen.

Answer: (a) $2 \times 3^2 \times 11$ (b)(i) 12 (b)(ii) 90

Question 13

Many candidates found the correct answers for x and y . Those who did not, seemed to have confused congruent with similar.

Answers for z were more frequently incorrect, with 102, from $180 - 78$; or 78 being the usual wrong values. The sum of the interior angles was sometimes seen to be equated to 360° and it was often in the calculation of $360 - 78 - 77 - 90$ that careless slips were made.

Answer: $x = 45$ $y = 20$ $z = 115$

Question 14

- (a) Most candidates answered this part correctly. A few gave 320, from 80×4 .
- (b) Those who knew that the average speed is calculated from $\frac{\text{total distance travelled}}{\text{total time taken}}$ were usually able to answer this part correctly. Some started by dividing 25 into the total distance, 125 km, to obtain the total time of 5 hours. Others solved the equation $\frac{80 + 45}{4 + t} = 25$.

Many used wrong methods such as one of the following:

Assuming that the speed from Q to R is the difference between the speed from P to R and the speed from P to Q . This gave the speed from Q to R as $25 - 20 = 5$ km/h and a resulting time of 9 h from $\frac{45}{5}$; 1.8, from $\frac{9}{5}$.

Assuming the speed for the whole journey was the average of the speeds for the two parts. This gave a speed of 30 km/h for the second part and a resulting time of $\frac{45}{30} = 1.5$ h.

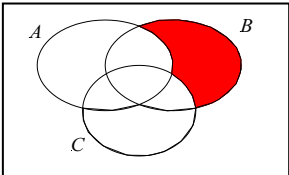
Taking the speed for the second part to be 25 km/h and obtaining a time of $\frac{45}{25} = 1.8$ h.

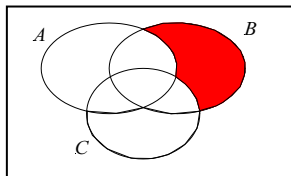
Assuming times are directly proportional, thus $80 = 4$ hours, $45 = x$ hours so $x = \frac{4 \times 45}{80} = 2.25$ h.

Answer: (a) 20 (b) 1

Question 15

- (a) Most candidates attempted this question, though there was a large variety of regions shaded.
- (b)(i) There were many sensible attempts at finding the elements of $W \cup H$. Some candidates gave these elements as their answer, not knowing that $n(W \cup H)$ means the *number* of elements in the set. Others found $n(W \cap H)$.
- (ii) Often answered correctly. The usual sensible wrong answers were to include 12 or 15.

Answer: (a)  (b)(i) 6 (b)(ii) 10, 14, 16



Question 16

- (a)(i) Many realised that this part involved the “difference of squares” and gave the correct factors. Common wrong answers were $(2p - 3q)^2$; $(4p + 3q)(4p - 3q)$; $(4p + 9q)(4p - 9q)$.
- (ii) There were many correct answers. Common wrong answers were $(2n + 1)(n - 3)$; $(2n + 3)(n - 1)$; $(2n - 3)(n + 1)$.
- (b) Most candidates obtained the correct fraction, though some tried to simplify it to fractions such as $\frac{17}{12xy}$; $\frac{17yx}{12xy}$; $\frac{17}{12}$. Weaker candidates simply added the numerator terms and denominator terms separately to obtain an answer such as $\frac{5}{4x+3y}$.

Answer: (a)(i) $(2p - 3q)(2p + 3q)$ (a)(ii) $(2n - 1)(n + 3)$ (b) $\frac{9y + 8x}{12xy}$

Question 17

Throughout this question problems arose when candidates assumed pairs of lines to be parallel which were not. Part (b) was the most successfully answered and part (c) the least.

Some candidates obtained answers that they should have realised were clearly the wrong size just by looking at the diagram.

- (a) Those candidates who applied the ‘tangent-radius’ result that angle $OCT = 90^\circ$ obtained the correct answer. The usual wrong answer was 48.
- (b) This part was usually answered correctly. Common wrong answers were 48; 132.
- (c) Those candidates who correctly applied the ‘opposite angles in a cyclic quadrilateral’ property obtained the correct answer. The usual wrong answers were 70; 110, seemingly obtained by assuming that the quadrilateral formed by E, D, C and the point of intersection of EB and AC is either a parallelogram or a cyclic quadrilateral.

Answer: (a) 28° (b) 62° (c) 48°

Question 18

- (a) There were many reasonable attempts to this part. Some candidates need to be made aware that the inequalities which define the region *inside* a triangle are strict inequalities, and must not include equality as well.
- (b) Those candidates who attempted this part sensibly, usually found the coordinates, (4, 4.5), where the line $x = 4$ intersected the line $y = x + \frac{1}{2}$. Unfortunately, many gave $k = 4.5$ as their answer, not realising that this point does not lie *inside* the triangle and that the question requested an *integer* value for k .

Answer: (a) $x > 3$; $y < 6$; $y > x + \frac{1}{2}$ (b) 5

Question 19

Most candidates omitted this question, or else tried to use the result

$$\frac{180 \times (n - 2)}{n} = \text{An angle},$$

which only applies to *regular* polygons, and not to the *irregular* one in this question. The majority of candidates who attempted this question seemed to have trouble in relating the number of sides in the polygon to the number of 155° and 140° angles.

Those candidates who used the result that

the sum of the angles in an n -sided polygon is $180 \times (n - 2)$ degrees

were more likely to do work of some merit.

Answer: 12

Question 20

- (a) (i) Most candidates seemed to know how to attempt this part. The horizontal scale was sometimes misread and the answer 65.2 given. A few gave 65, being the middle value of 60 to 70.
- (ii) Some candidates seemed unfamiliar with percentiles. The answer 61.6 from a cumulative frequency of 30, and not the 30th percentile, was sometimes seen.
- (iii) Many candidates attempted this part correctly. The usual incorrect answers were 240; 150; 60, by subtracting 240 from 400 incorrectly.
- (b) There were many successful answers to this part. The usual errors were to draw a curve 0.2 or 0.4 kg on either side of the given curve; to draw a curve from (58, 0) to (68, 400); to draw what appeared to be a correct curve but only taking it to (70, 340) or similar; to extend the curve to (72, 450). A few drew a line from (70, 400) to (72, 400). Others omitted this part.

Answer: (a)(i) 65.4 (a)(ii) 64 (a)(iii) 160 (b) Parallel curve from (62, 0) to (72, 400)

Question 21

This question was quite often omitted, perhaps because of a lack of geometrical instruments, or perhaps because many candidates need to get a better understanding of loci. Those who understand loci usually did quite well, drawing the required loci to an acceptable degree of accuracy. Occasionally there was confusion between perpendicular bisectors of sides and bisectors of angles.

Some candidates need to be made aware that, when the question requests the locus of points inside the triangle, each locus should cross the whole triangle.

- (a) Many candidates measured the correct angle and gave an acceptable bearing. Some gave a bearing of approximately 083° . A few measured the distance of C from A .
- (b)(i) Most of the candidates who attempted this part gave an acceptable answer. A few constructed a line that did not cross the whole triangle.
- (ii) Most of the candidates who attempted this part gave an acceptable answer. Some bisected the wrong angle, or else constructed a perpendicular bisector.
- (c) Those who constructed acceptable loci in part (b) usually realised that D is at the point of intersection and found the actual distance of D from A . The most common error arose from failing to convert the measured length into kilometres giving answers such as 41 km; 8.2 km; 4.1 km.

Answer: (a) 096° to 098° (b)(i) Perpendicular bisector of BC (b)(ii) Bisector of angle ABC (c) 80 to 84 km.

Question 22

Many candidates demonstrated a good understanding of coordinates and equations of straight lines.

- (a) Usually answered well. The most common error was to give the y coordinate as $\frac{1}{2}$. Other errors were to use an incorrect result for the mid-point; to make an error in evaluating correct expressions.
- (b) Many candidates knew how to find the gradient. Sometimes arithmetic errors such as $2 - (-3) = 6$ and $7 - 1 = 8$ occurred. Occasionally the reciprocal value, $\frac{6}{5}$ or 1.2, was obtained.
- (c)(i) Many candidates substituted the coordinates of Q into the equation of the line L and obtained the value of k correctly. A few substituted the coordinates of P or else gave $k = 2x - 5y$ as their answer.
- (ii) Many candidates substituted the coordinates of Q into the equation $x + Ay = 3$ to get the wrong answer of $A = -2$. Successful candidates compared the equation $x + Ay = 3$ with the equation of L , most easily by converting L to $x - \frac{5}{2}y = \frac{k}{2}$.

Answer: (a) $(4, -\frac{1}{2})$ (b) $\frac{5}{6}$ (c)(i) 4 (c)(ii) -2.5

Question 23

Answers to this question showed that many candidates need to get a better understanding of probability and the manner in which probabilities of combined events are calculated. Many answers to probabilities had values greater than 1.

- (a) Only a minority of candidates answered this part correctly. Many did not seem to recognise that probabilities were required. Common wrong answers were whole numbers, such as 1, 2, 3, 4 or 2, 4, 6, 8; fractions that are greater than 1; $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}$.

- (b)(i)** This part was almost always answered correctly.
- (ii)** More able candidates obtained the correct probabilities from their table in the previous part. Occasionally $\frac{1}{16}$ was given instead of 0. Others seemed to be looking for patterns in the fractions and gave answers such as $\frac{13}{18}, \frac{11}{20}, \frac{9}{22}$.
- (c)** Many candidates omitted this part. A few realised that it was necessary, first of all, to find the probability of combined events such as 'Anil scores 2' and 'Billie scores more than 2' by multiplying the first entry in each of the tables in part **(a)** and part **(b)(ii)**. This process had to be repeated for the other entries and the resulting probabilities added together. Common wrong methods were to subtract these entries; to multiply probabilities taken only from part **(b)(ii)**.

Answer. **(a)** $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$ **(b)(i)** 5 6 7 8 **(b)(ii)** $\left(\frac{15}{16}\right) \frac{10}{16} \frac{3}{16} 0$ **(c)** $\frac{7}{16}$

Question 24

- (a)** Generally answered well, though some candidates, often having written down the next few terms at the beginning of the question, only gave the next term as their answer and not the next two terms requested in the question.
- (b)** Those candidates who realised that the terms ended either in a 3, or in a 7, wrote down the correct answer. Many did not seem to realise this and gave answers such as 1000, 998, 995, 993. A few confused the number of the term (198) with its value.
- (c)** A surprising number of candidates did not notice that, with the following terms being found by alternately adding 4 and 6 to the previous term, any two terms being positioned two apart will have values that differ by 4 plus 6, which equals 10. Common wrong answers were 92; 2.
- (d)** A few candidates noticed that all the even numbered terms end in a 7 and the number in front of the 7 is half the term number. Some used an equivalent $5n + 7$ for the even numbered terms. Others tried to use the result n^{th} term is $a + (n - 1)d$, but with $d = 4$ or with $d = 6$; or attempted to use proportionality, such as first term is 13, so 80th term is 13×80 . Many did not attempt this part.
- (e)** A few candidates noticed that a term ending in a 3 is an odd-numbered term, and the number in front of the 3 (here 20) is the 20th odd term. Further, there are $20 - 1$ even-numbered terms before the 20th odd term. Some used $5n + 7$ as the even numbered terms and $5n + 7 = 197$, the term before 203. Others used a result such as the r th odd term is given by $10r + 3$. Most candidates attempted to use inappropriate methods as in the previous part, or else omitted this part.

Answer. **(a)** 43 47 **(b)** 997 **(c)** 10 **(d)** 407 **(e)** 39

Question 25

Attempts at this question showed that some candidates need to get a better understanding of the properties of a speed-time graph, and to appreciate that the "D-S-T triangle" is not a valid method when there is an acceleration or retardation.

- (a)** Many candidates did not realise that the acceleration is the gradient of the line joining (0, 0) to (10, 15). Some candidates confused velocity for acceleration. There were many wrong answers. For example
10 is 1.5, so 7 is $7 \times 0.15 = 1.05$;
 $15 \times 7 = 105$, then $10x = 105$, so $x = 10.5$;
gradient = $\frac{10 - 0}{7 - 0} = \frac{10}{7}$.

- (b) Of those who attempted this part, many recognised that it was necessary to find the area under the graph. However, instead of $k - 10$ some used k or $10 - k$ as one of the lengths. It was not uncommon to see an unsimplified expression, instead of the simplified one requested in the question. Some, having obtained $15k - 75$, went on to give $k - 5$, or $k = 5$, as their answer. A few gave an answer that involved an inequality.
- (c) (i) There were some correct graphs. Many who attempted this part drew a line that was part dotted, part solid; starting at the sloping line given or at the end of their drawn sloping line, instead of starting at $(0, 12)$. A large number of candidates did not attempt this part.
- (ii) Very few candidates made a sensible attempt at this part by equating $12k$ to their distance expression from part (b).

Answer: (a) 1.5 (b) $15k - 75$ (c)(i) Horizontal line from $(0, 12)$ (c)(ii) 25

Question 26

- (a) There were many good attempts at this part, with a completely correct answer, or one with incorrect elements arising from arithmetic slips. There was also a variety of matrices of the wrong size given.
- (b) (i) Most candidates showed that they were familiar with an inverse matrix. Errors sometimes occurred in calculating the determinant, or in omitting it altogether.
- (ii) Many candidates did not attempt this part, or offered what often seemed to be a random mix of matrix products. More sensible attempts were made by candidates who realised that the transformations were related by $T_3(T_2(A)) = T_1(A)$ and were also able to put this relationship into a matrix equation such as

$$\mathbf{M} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix} \text{ or } \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix} = \text{the answer to part (a)}.$$

Answer: (a) $\begin{pmatrix} 2 & 2 & 8 \\ 0 & 1 & 3 \end{pmatrix}$ (b)(i) $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$ (b)(ii) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

MATHEMATICS D

Paper 4024/21

Paper 2

Key Messages

- To succeed in this paper, candidates need to have completed full syllabus coverage and remember the necessary formulae and apply them appropriately.
- Candidates should show all working clearly and use a suitable degree of accuracy in their working. Final answers should be rounded correct to three significant figures where appropriate.
- Where candidates are asked to show a result, all stages of their working leading to this final result should be clearly shown.

General Comments

The paper consisted of questions ranging from routine tasks to questions requiring candidates to interpret information and to solve problems which provided challenge for the more able candidates. Scripts covering the whole mark range were seen, demonstrating that the paper discriminated well. Some candidates did not attempt a large number of the questions which suggested that they had not covered all of the syllabus content.

Working was often clearly set out with all stages of calculations shown. In algebra questions, candidates did not always use brackets correctly, which led to incorrect answers. In mensuration questions, there was some confusion seen between formulae and it was common to see a volume formula used where an area was required, area where volume was required or area where perimeter was required. In several places candidates incorrectly assumed that angles were right angles and used trigonometry with a right-angled triangle where the sine or cosine rule was required. Some candidates used a value of $\frac{22}{7}$ for π when they should use either their calculator value or 3.142.

Comments on Specific Questions

Section A

Question 1

- (a) Many candidates reached the correct answer. A common incorrect answer was £230 which was the total interest received in 4 years rather than the amount of money in the account at the end of that period.
- (b) This part was often answered correctly. Some candidates rounded their final answer to \$263 which was not appropriate as \$262.50 is an exact answer.
- (c) The values of w and x were usually calculated correctly. In order to calculate the value of y , candidates were then required to perform a reverse percentage calculation using the total of \$32.40 and the 20% given in the question. It was common to see candidates finding 120% of \$32.40, giving a value of \$38.88, or using 20% in a reverse percentage calculation, giving a value of \$162. Correct values of y were uncommon. Many candidates used their incorrect value of y correctly to find a value for z .

Answers: (a) 2730; (b) 262.50; (c) $w = 4.65$, $x = 0.75$, $y = 40.50$, $z = 31.35$

Question 2

- (a) (i) Many candidates recognised that they should use Pythagoras' theorem to find AB , however many could not use the information in the question to identify that the height of the required triangle was 12 cm. It was common to see 11 cm used, from halving the height of the pentagon, or 10 cm used from assuming that BC was equal to the height of the triangle.
- (ii) Trigonometry was generally used in this part to find angle ABE , using the same side lengths as in the previous part of the question. Some answers of 38.7° were seen where candidates had not added 90° to their angle ABE to get the required angle ABC . Some candidates tried to use the sum of the angles in a pentagon to calculate angle ABC , which was not successful as angle BAE was not known.
- (b) Candidates usually identified that the sine rule was required to calculate the required angle and it was usually applied correctly. Some inaccurate final answers were seen resulting from premature approximation of the value of $\sin 65$. Candidates should be advised to rearrange the sine rule formula before carrying out any calculations to avoid this error. Some candidates attempted to apply the cosine rule or assumed that this was a right-angled triangle. If the triangle did include a right angle, this would be labelled on the diagram.

Answers: (a)(i) 19.2; (ii) 128.7; (b) 44.8

Question 3

- (a) (i) Most candidates attempted the correct process of finding the matrix $2\mathbf{A}$ first then subtracting matrix \mathbf{B} . In general, errors were due to incorrect subtraction of negative numbers.
- (ii) Most candidates attempted the correct process of finding the inverse of the given matrix. The determinant was often correct but errors with the negative signs or in the positioning of the numbers in the inverse matrix were common.
- (b) Candidates found the manipulation required to rearrange this equation difficult. Errors in negative signs were common. Those candidates who correctly reached an equation such as $2\mathbf{C} = -4 \begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}$ did not always understand how they could rearrange this to find matrix \mathbf{C} . It was common to see candidates dividing terms when they should be multiplying them.
- (c) (i) Correct answers to this part were common. Some candidates did not add the two terms in each row and gave a 3 by 2 matrix as their answer.
- (ii) Candidates usually referred to an amount of money in their answer, but they did not always state that it was the amount per week. Some were confused about whether it was an amount per kilogram or the number of kilograms of fruit.
- (iii) Most candidates correctly added the values to give an amount of money, but few converted the amount from cents to dollars. Some candidates rounded the amount to 85.8 which was not appropriate as \$85.75 is an exact answer.

Answers: (a)(i) $\begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix}$; (ii) $\frac{1}{4} \begin{pmatrix} 2 & -2 \\ 3 & -1 \end{pmatrix}$; (b) $\begin{pmatrix} 4 & -2 \\ 0 & -6 \end{pmatrix}$; (c)(i) $\begin{pmatrix} 3110 \\ 2715 \\ 2750 \end{pmatrix}$; (ii) Amount in cents for each week;
(iii) 85.75

Question 4

- (a) Many correct answers were seen. Some candidates' answers indicated that they had confused the symbols for union and intersection.
- (b) Candidates found this part very difficult and few correct answers were seen. Those candidates who showed some understanding of set notation usually included the complement symbol in their

response. They did not always use the union and intersection symbol correctly or understand the significance of where the brackets should be positioned.

- (c) (i) Many partially correct Venn diagrams were seen. The most common errors were in the positioning of the 1 and 9. Many candidates did not identify that 1 was a square number. The 9 and 1 were often placed in more than one position on the diagram. In some cases the 2, 6, 8 and 10 were omitted or incorrectly positioned in Q .
- (ii) Many candidates used their diagram to find the number of elements in the required set. Some candidates listed all of the elements in the set rather than stating the number of elements.
- (iii) Probabilities were usually given as fractions with a denominator of 10. It was common for candidates to give the probability that m was a member of Q' rather than a member of $P \cap Q'$.

Answers: (a) Correct shading; (b) $E \cap (D \cup F)$; (c)(i) 10 numbers positioned correctly; (ii) 7; (iii) $\frac{3}{10}$

Question 5

- (a) Many candidates factorised the expression completely. Partial factorisations to $3xy(2xy^2 - 5x^2)$ were also common.
- (b) Most candidates correctly eliminated the fractions and reached the equation $4x + 8 + 2x = 3x^2 + 6x$ and often rearranged this correctly to $3x^2 - 8 = 0$. Having reached this quadratic equation, many candidates could not solve it.
- (c) (i) The lines $x = 1$ and $y = 4$ were often drawn correctly on the diagram. Candidates had more difficulty in drawing the lines $x + y = 6$ and $y = x$ and in some cases one or other of these lines was omitted. Where all four lines had been drawn, the correct region was usually identified.
- (ii) A calculation using $\frac{y_2 - y_1}{x_2 - x_1}$ was often seen in this part, but (6, 6) was often used for the coordinates of N rather than the correct coordinates, (3, 3). In some cases an incorrect answer of -2 resulted from calculation of the gradient as (change in x) \div (change in y).

Answers: (a) $3x^2y(2y^2 - 5x)$; (b) $x = \pm 1.63$; (c)(i) Correct region identified; (ii) $-\frac{1}{2}$

Question 6

- (a) (i) Although many candidates identified that the moves were 1 across and 3 down to get from Q to R the values were often reversed and signs were often incorrect. Some candidates found \overrightarrow{PR} .
- (ii) This part was generally answered correctly.
- (b) Candidates struggle with vectors and few correct answers were seen in any of part (b). Candidates who showed vector expressions on the diagram often made more progress with the question, but several stages of working were required to find vector CE and many made errors with signs and simplification so did not reach the correct expression for CE . After an incorrect expression for CE , candidates rarely showed any further correct work.

Answers: (a)(i) $a = 1$, $b = -3$; (ii) 5.39; (b)(i) $\mathbf{b} - \frac{1}{2}\mathbf{a}$; (ii) $2\mathbf{b} + \frac{1}{2}\mathbf{a}$; (iii) 1 : 3

Section B

Question 7

- (a) Few candidates drew a correct quadrilateral meeting the conditions in the question. Most attempts were quadrilaterals with a vertical line of symmetry or a quadrilateral with rotational symmetry order 2 but no line of symmetry.
- (b) Few candidates drew a shape meeting the conditions in the question. Common incorrect answers were a rhombus with centre M , which has rotational symmetry order 2, or a square of side 2 cm with M at one vertex.
- (c) Those candidates who attempted this question often drew a correct triangle in part (i) and stated a correct equation in part (ii). They had more difficulty in part (iii) where the transformation was not always identified as a translation, and when it was, the vector was not always correct. Some candidates drew triangle D on the grid and gained partial credit.
- (iv) Few candidates stated the correct coordinates for triangle E and hence could not describe the transformation or state the correct matrix for the transformation. Many candidates had used the line $x = 0$ for the reflection rather than $y = 0$. When candidates attempted to describe a rotation in part (b), they usually gave a full description including the angle, direction and centre of rotation. Very few correct matrices were seen in part (c).

Answers: (a) Correct quadrilateral; (b) Correct shape; (c)(i) C at (3, 1), (3, 3), (4, 3); (ii) $y = x$;

(iii) Translation $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$; (iv)(a) (2, 0), (4, 0), (4, -1); (b) Rotation 90° , clockwise, centre (0, 0);

(c) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Question 8

- (a) When candidates are asked to show a result, they are expected to start from the known formulae and work towards the required result. In this case, they needed to identify that the total surface area of the cone is given by $\pi r^2 + \pi rl$ and then find an expression for l in terms of r from the information given in the question. By using the substitution $l = r + 4$, they could then manipulate their expression for surface area to give the required result. Some candidates showed very clear stages of working to reach this result. Others made errors or attempted to work backwards from the given result which was not acceptable. Some candidates used $l = 4r$ in place of $l = r + 4$.
- (b) This part was generally answered correctly.
- (c) The points were often plotted correctly and joined with a smooth curve. Some inaccurate plots were seen, in particular at (6, 288).
- (d) Many candidates found this part difficult. It was common to see attempts using the formula $A = 6r(r + 2)$ from part (a) which did not lead to an expression for h . Where candidates did attempt to use Pythagoras' theorem, they often used h , rather than l , as the hypotenuse.
- (e) Candidates who had the correct expression for h usually reached the correct radius in this part. Where candidates had an incorrect expression for h in terms of r , they often substituted correctly for h in an attempt to find the radius.
- (f) (i) Most candidates read the correct value from their graph. Rounding an accurate reading to an answer of 5 cm was not accepted.
- (ii) Many candidates omitted this part of the question, however some did realise that they needed to use the value of r they had read from the graph with their expression for h to find the height of the box. Correct answers to this part were rare, because few realised the significance of the requirement for an integer answer. Some candidates rounded their answer down to 7, which was less than the height of the cone so was not acceptable.

Answers: (a) $A = 6r(r + 2)$ correctly derived; (b) 48, 90; (c) Correct curve; (d) $\sqrt{8r + 16}$; (e) 16; (f)(i) 4.9;
(ii) 8

Question 9

- (a) (i) Many candidates attempted to read the median at the correct point on the graph, although some did not read the scale on the time axis accurately.
- (ii) Many candidates attempted to read the upper and lower quartiles from the graph, but again these were not always read accurately. Some candidates identified that the upper and lower quartiles were at cumulative frequencies of 12.5 and 37.5 but they then subtracted these values to get 25 and read the value for the time using this cumulative frequency thus giving the median for their answer in this part.
- (b) Some correct frequency tables were seen. Common errors were to give the cumulative frequencies or to use the values of 3 and 5 in the table as the first terms of a sequence, leading to the incorrect values 7, 9, 11, 13, 15.
- (c) Very few candidates could interpret what was required in this part and if they did select 3 track lengths to add, these were rarely 6, 6 and 5:30. Some candidates did not read the question carefully and attempted to find the total length of all 50 tracks.
- (d) (i) Candidates found it difficult to interpret the timetable and many did not understand that the journey started at 09 58 and finished at 10 21. It was common to see answers such as 18 minutes, from using 10 03 as the start time, or 18 minutes from using 10 16 as the finish time. Some candidates thought that 10 22 was missing from the last cell in the table and gave an answer of 24 minutes. An answer of 63 minutes was common where candidates had used 100 minutes in an hour.
- (ii) Some candidates worked out that the journey took 5 minutes, but it was also common to see a journey time of 6 minutes. Some attempted to use times in their probability rather than the number of tracks. Answers that were fractions with a denominator of 50 were rarely seen. Some candidates did not understand that a probability must be less than or equal to 1.
- (e) In this part, candidates had the same difficulty with probability as in the previous part, so correct answers were rarely seen. Very few candidates realised the need to multiply two probabilities to get their answer, and those that did seldom realised that, as the two tracks were different, the denominators of the fractions would be 50 and 49 rather than 50 and 50.

Answers: (a)(i) 4 minutes 18 seconds; (ii) 1 minute 0 seconds; (b) 10, 12, 13, 5, 2;
(c) 17 minutes 30 seconds; (d)(i) 23; (ii) $\frac{7}{50}$; (e) $\frac{4}{175}$

Question 10

- (a) (i) Candidates were expected to use the formula for the area of a trapezium together with the information given in the question to show the required result. They needed to start by identifying that the perpendicular height of the trapezium was $(x - 3)$. Some candidates started with a correct expression and showed clear correct stages of working leading to the required equation. In other cases, working was hard to follow and candidates were penalised for missing brackets and incorrect stages of working. Some candidates attempted to solve the given quadratic equation in this part.
- (ii) In this part candidates had to solve the given quadratic equation. They are required to show correct substitution into the quadratic formula to gain full credit. Some candidates did not give their answer correct to 1 decimal place as instructed in the question.
- (iii) Although many candidates attempted to set up an equation for the perimeter of the trapezium, many did not include all four sides. Some candidates did not use the information that AD was 0.8 cm longer than BC , so had an equation with two unknowns that they could not solve. In some cases candidates found the length of BC rather than AD . There was some confusion seen between area and perimeter, and in some cases the formula for the area of a trapezium was used in this part.

- (b)(i)** In this part it was clear that many candidates had not read the question carefully, or did not understand the term reflex angle, and they used 252° as the interior angle at LMN . It was then common to subtract this from 360° to give the answer 108° as the angle MNO . Those candidates who understood the meaning of reflex angle LMN usually went on to find the interior angle as 108° and then correctly found angle MNO .
- (ii)** Many candidates did not realise that they needed to find angle OLM in the trapezium in order to answer this question. It was common to see manipulation of the ratio $1 : 2$ given in the question and answers such as $\frac{1}{2}$ with no reference to the angles found in the previous part. Candidates who found angle OLM as 126° usually went on to get the correct answer.

Answers: **(a)(i)** $x^2 + 12x - 195 = 0$ correctly derived; **(ii)** 9.2; **(iii)** 7.3; **(b)(i)** 72° ; **(ii)** $\frac{4}{7}$

Question 11

- (a)(i)** Many candidates used the correct formula and reached the correct result in this part. It was clear that some candidates did not understand what was meant by area of cross-section, as they calculated the volume of the prism. Some candidates assumed that the triangle was right-angled.
- (ii)** All that was required in this part was for candidates to multiply the cross-sectional area from the previous part by 20 to give the volume of the prism. Many candidates used a completely new calculation, and formulae for the volume of a cuboid or pyramid were often seen here.
- (iii)** The first step required here was to use the cosine rule to find the third side in the triangle, but few candidates realised this. Those that did identify this requirement usually calculated the length correctly. When calculating the surface area of the prism most candidates included the two triangular faces but some omitted one of the rectangular faces. Some candidates appeared to assume that two of the sides of the triangle were the same but did not state this explicitly.
- (b)** Many candidates used the correct formula for the volume of a cylinder here, although formulae such as πrh and $\frac{1}{3}\pi r^2 h$ were also used. It was common to see 70 and 0.1 substituted into the formula and a correct rearrangement to reach an expression for r^2 . Few candidates however identified that the units given in the question were different and that a conversion was required. Some candidates did convert 70 cm to 0.7 m, but did not read the question carefully and gave their answer in metres rather than centimetres, as required. It should be noted that candidates are expected to use their calculator value of π or 3.142 on this paper and those that used the value $\frac{22}{7}$ reached an inaccurate final answer.

Answers: **(a)(i)** 9.19; **(ii)** 184; **(iii)** 310; **(b)** 21.3

MATHEMATICS D

Paper 4024/22

Paper 2

Key Messages

To succeed in this paper, candidates need to have completed full syllabus coverage and should clearly show all working. Numerical working should in general maintain 3 significant figure accuracy.

General Comments

Scripts covering the whole mark range were seen. The wide variety of different types of question gave candidates of all abilities the opportunity to demonstrate what they knew. When questions were omitted, sometimes in significant numbers on individual scripts, it seemed likely that the candidates involved had not covered all of the relevant syllabus content.

Working was often clearly set out, and in general, the standard of written work and presentation was good. In numerical work, all stages of a calculation were generally shown. Very few answers appeared with no working at all, but some candidates show very little by way of working. Method marks cannot be awarded without seeing the relevant evidence. Where candidates are asked to show a result, all stages of their working leading to this final result should be clearly shown. There are still some candidates who could be encouraged to write more clearly and to set out their work in a more orderly fashion. Clear organisation of work is also particularly important when devising strategies to solve problems. Candidates could be encouraged to be clearer in explaining the steps in their work.

In numerical questions, marks were sometimes lost through premature approximation. Generally, final answers need to be correct to 3 significant figures. This means working occasionally with more significant figures during the course of a calculation.

In the algebra questions, many candidates continue to gain good marks when factorising standard expressions, solving simultaneous equations, solving inequalities and finding the roots of a quadratic equation. Situations involving negative signs and the reorganisation of equations, particularly when there are fractional terms, always require the utmost care.

Candidates continue to do well in trigonometry when the cosine and sine rules are required. There is a tendency when straight forward right-angled triangle trigonometry would be appropriate to make calculations unnecessarily complicated by using these formulae more suited to non-right-angled triangles. These situations often lead to loss of marks through premature approximation. Also, in trigonometry, candidates continue to assume that certain angles in diagrams are right angles.

The mensuration section of the syllabus highlights a particular problem, seen often this year, involving the use of formulae. The syllabus requires certain formulae to be remembered. In other cases, the formula required will be given as part of a particular question. It is important that candidates know exactly which formulae must be remembered. Instances of poorly remembered formulae included for example the use of a volume formula when an area was required, an area formula when a volume was required or an area formula when a length was required. Instances of using the formula given in a question incorrectly included using the formula for the surface area of a sphere to calculate the area of a circle when clearly the formula for the area of a circle could not be remembered.

There was some confident work with vectors and transformations.

The responses in the question involving statistics and probability revealed some significant misunderstandings.

Comments on Specific Questions

Section A

Question 1

- (a) (i)(a) This question was generally well answered. In nearly every case, the separate parts of 60% of \$360 and $12 \times \$15$ were appreciated but they were not always correctly combined.
- (b) This part presented no problem to candidates who were able to accept an answer greater than 100%. Those working correctly often preferred to give the answer as 10%, the percentage increase from \$360 to \$396. Those working incorrectly evaluated $\frac{360}{396}$ as a percentage.
- (ii) Many candidates find reverse percentages difficult. It did seem this year that more candidates were confident with the arithmetic required. The common misunderstanding in purely arithmetical solutions was to calculate 26% of \$569.80 leading to the answer \$717.95. Candidates seem to be unwilling to consider this type of problem in terms of algebra, where an equation could be constructed from the information given in the question.
- (b) This question was often well done. Most solutions seen contained some correct work. Candidates could be encouraged to improve the layout of their work in problems such as this, giving some explanation of what they are doing.

Answers: (a)(i)(a) 396 (i)(b) 110 (ii) 770 (b) 1.21

Question 2

- (a) As stated in the question, a calculation was expected. Many candidates used $\frac{1}{2} \times \text{base} \times \text{height}$ successfully, either with or without the need to plot points on the grid. The determinant method was also used with success. A common error continues to be the misunderstanding that the area of a triangle is half the product of two sides. Calculations involving angles measured on the diagram were not accepted. Some candidates counted squares on the grid. Again, this was not accepted.
- (b) Again, the calculation using the coordinates of the relevant points was expected. Generally, this result is well known and was well handled. The measurement of AB on the diagram was not accepted.
- (c) This part proved difficult for many candidates. First of all, it was not always clear that the concept of perimeter was understood. Then, when three sides were added together, a common error was that one of them was 4, even after 4 had been used correctly in part (a) as the height of the triangle.
- (d) Many candidates did not use the tan ratio from the right-angled triangle as expected, but instead used the cosine rule. This made the task of obtaining an accurate result more difficult.

Answers: (a) 14 (b) 10.8 (c) 22.8 (d) 21.8

Question 3

- (a) (i) The justification here depends on establishing that \widehat{ABC} is a right angle. So it was essential to make reference to the angle in a semicircle. Many attempts at this question merely offered a calculation based on the sum of the angles in a triangle, assuming that \widehat{ABC} was 90° .
- (ii) A full solution here required the use of the isosceles triangle OCD . A number of candidates realised that $OC = 3$ but were unable to get any further. Others realised that $\widehat{OCD} = 124^\circ$ but were unable to complete correctly. Angles of 34° and 56° were seen used incorrectly, sometimes justified by non-existent parallel lines.

- (iii) Angles in the isosceles triangle OCE were required here. Some candidates were able to recover at this point. There were many incorrect assumptions seen in attempts to find this angle.
- (b) (i) There were very few complete solutions seen. It was clear in many cases that the candidate knew that what was required was to show that triangle RPT was isosceles, but all too often, no further work was attempted. Some candidates were able to pick out equal angles, usually \widehat{SPR} and \widehat{PRT} , often with the correct reason given, but only very few candidates could link all the necessary information required into a coherent explanation.
- (ii) The given ratios were converted into an equation that could be solved by a good many candidates. Many managed only the incorrect $\frac{12}{5} = \frac{8.5}{x}$. Some candidates understood the situation well and were able to arrive at the correct answer by inspection. Candidates embarking on algebraic solutions to problems using x could be encouraged to state what their x represents.

Answers: (a)(i) Convincing explanation (ii) 28 (iii) 76 (b)(i) convincing explanation (ii) 2.5

Question 4

- (a) (i) The formula given in the question for the volume of a sphere had to be adjusted here to deal with a hemisphere. Many candidates overlooked this point. A number of equations using r^3 initially were transformed correctly but were then evaluated using the square root function rather than the cube root function. Substitution of $r = 5$ or 20 in the volume formula was seen. Some candidates used a completely different formula for the volume. Some correct work was spoilt by not giving the final answer correct to 3 significant figures.
- (ii) It was expected that candidates would compare the volumes of geometrically similar figures. There were also some accurate solutions using longer methods. A common misunderstanding was to assume that the two lengths were in the same ratio as the volumes.
- (b) This part required the correct use of the formula given in the question and also the correct use of the formula for the area of a circle, not given in the question. Candidates need to be aware that some formulae required in mensuration questions must be remembered. Full marks in this question also depended upon the ability to put together a strategy that took full account of all the information given in the question. There were candidates able to do this. As in part (a)(i), a common error was the failure to adapt the given formula to the hemisphere described in the question. Another common error was to use this formula to deal with the area of a circular hole. Some solutions used 50×1.5 as the total circular area removed from the surface of the hemisphere. The basic subtraction required was usually appreciated, but adjusting the units involved proved another hurdle for many candidates.

Answers: (a)(i) 2.12 (ii) 6.79 (b) 187

Question 5

- (a) Pythagoras was usually seen as the appropriate strategy for this question. There were some who added the squares. There were some attempts using inappropriate strategies for which dimensions had to be made up because they were not given as part of the question. Candidates should realise that all the information they need has been given in the question.
- (b) Again, this part was generally well done. Some candidates used \cos or \tan instead of \sin . The use of $AE = 30$ instead of EF was seen. The sine rule was used effectively in some cases. In general, this is more appropriate for a general triangle.
- (c) Often completely correct. Again, reading the question carefully was essential. In some solutions, the angle of 30° was shown with the horizontal.

Answers: (a) 51.2 (b) 12.7 (c) 40.4

Question 6

- (a) (i) Candidates generally appreciated the need to solve two linear equations. Complete solutions were seen with both answers corrected as required. Some marks were lost for incorrect rounding and for sign and numerical errors. Very commonly, candidates interpreted $\frac{\sqrt{5}}{2}$ as $\sqrt{\frac{5}{2}}$. Some candidates, having found one value, gave as their answer positive and negative versions of this value.
- (ii) It was expected that candidates would square both sides of the given equation and compare the outcome with $x^2 + Bx + C = 0$. Only a minority of candidates did this. Some candidates showed their understanding of the situation by using their decimal answers from the previous part to generate two simultaneous equations, some of which were successfully solved. A number of candidates spotted that B must be 7, but were unable to find C .
- (b) There were many careful solutions of this inequality, particularly by those who left $-3x$ on one side of the inequality and who therefore had to adjust to $+x$ later on.
- (c) Much careful and accurate work was seen here. Nearly all candidates understand what is required with this type of expression. Not all candidates managed to negotiate the negative signs correctly. Probably the initial reordering as $6x - xt + 18y - 3yt$ was more helpful in this regard.
- (d) Again, the majority of candidates understand what is required to solve simultaneous equations. The most successful strategy used was to equalise one set of coefficients. This seems to produce fewer problems when handling equations than the method of substitution, which introduces the further task of removing fractions. Very rarely, the matrix method was seen.

Answers: (a)(i) -4.62 -2.38 (ii) ($B =$) 7 ($C =$) 11 (b) $x < -2$ (c) $(x + 3y)(6 - t)$ (d) ($a =$) 17 ($b =$) -16

Section B

Question 7

- (a) It was not necessary to use expressions in x at this stage. Candidates who used the area sine formula directly were generally successful in showing that the first fraction given in the question was equal to the second. Of those who converted to algebraic notation at this stage, there were some convincing solutions, but often it was not clear that both sides of the equation had been fully considered. Many of the algebraic attempts did not at any point use the area sine formula.
- (b) Candidates who had begun the algebra in the previous part often managed to complete this part successfully. Sometimes candidates did not use brackets when converting to expressions in x . An expression such as $2x - 5 + 4$, for example, left without appropriate brackets, often led to later errors in the working. It was common to see an algebraic conversion left without equating this to $\frac{1}{3}$ in order to establish the equation.
- (c) (i) Candidates generally handled the formula correctly. Care was needed in dealing with the negative terms.
- (ii) Since both roots of the given equation were positive, it was necessary to show that one value was inappropriate because it would give a negative value for one of the sides. There were some correct answers. The answer of 0.33 with the reason that it was too small was not accepted.
- (d) Many candidates recognised the cosine rule here and evaluated it accurately. Some candidates left their result as an expression in x . Some inappropriate calculations were seen, including the use of a right angle at B .

Answers: (a) Correctly shown (b) $2x^2 - 19x + 6 = 0$ correctly obtained (c)(i) 9.17 0.33 (ii) 0.33 with reason (d) 6.35

Question 8

- (a) (i) Candidates usually remembered the correct formula here so there were some good, accurate solutions. Occasionally the formula for the area of a sector was used.
- (ii) Again the correct formula was often remembered. An error seen from time to time was the inclusion of $\sin 25$.
- (b) (i) Candidates did not always notice that this part carried on from the previous part.
- (ii) Again, links with previous parts could be carried through and these were noticed by some candidates. There were many fresh starts seen. Many candidates were aware of the rectangular faces and the top and bottom of the slice of cheese. The curved face caused problems for some. While most candidates picked up some credit in this question, only those who could organise their work clearly managed to work through to a successful conclusion.
- (iii) The successful solutions here were usually those where a clear sketch had been drawn or where there was a clear statement of the new radius and height. Again, fresh calculations seemed to be more common than adapting previous results.
- (c) (i) The formula for the volume of a cylinder is another result that the syllabus requires candidates to remember. The response to this question showed that this is not always the case.
- (ii) There were very few correct responses here. The connection with variation, one quantity varying inversely as the square of the other, was generally missed.

Answers: (a)(i) 2.62 (ii) 7.85 to 7.86 (b)(i) 39.3 (ii) 88.8 (iii) 471 to 472 (c)(i) $(h =) \frac{800}{\pi r^2}$ (ii) h is divided by 4

Question 9

- (a) The table was usually completed correctly. Sometimes the value was incorrectly rounded down to 35.
- (b) Points were usually plotted accurately resulting in acceptable curves. Sometimes the point (1, 29) was not joined to (1.5, 14).
- (c) (i) The concept of tangent was understood with acceptable attempts generally made to draw the required tangent accurately. Candidates generally showed their working for the calculation of the gradient, so marks could be given for $\frac{\delta y}{\delta x}$ clearly obtained from their tangent. Some tangents that were less well drawn led candidates to use adjacent points on the curve, such as (4, 8) and (4.5, 11) in their calculation of the gradient. This implied a chord rather than a tangent, so full credit could not be given. Also, when no tangent was drawn, no credit could be given in this question.
- (ii) Only a relatively few candidates realised that the gradient represented the speed of the moving object at that point. Descriptions using the idea of rate of change were usually spoiled by referring to the change of speed or even acceleration. Frequently, this part was left unanswered.
- (d) Most candidates answered this part well. Again, candidates often showed their working. It seemed that some candidates attempted to write down the answer directly from looking at the graph, without stating the relevant readings. It is not always clear where answers are coming from when this occurs. Common errors were to read off both values and omit the subtraction, or read off only one of the relevant values. Some candidates added 4 or 5 values together to obtain their answer.
- (e) (i) Following on from the previous part, the graph was usually read correctly. Identifying the direction of travel was more difficult. Many candidates got this right, while many had each direction reversed. Others were more confused, using such as "down" and "up", or even inserting numerical answers.
- (ii) A lot of candidates understood this part. Some candidates did not make the connection with the function given at the beginning of the question. The answer $d = 12$ was sometimes seen.

(iii) Correct equations in the previous part usually led to a successful result here.

Answers: (a) 36 (b) Correct plots and curve (c)(i) $4 < \text{gradient} < 6$ (ii) Speed (d) 2.5 (e)(i) 1.65 Towards 4.7
Away from (ii) $t^2 + \frac{48}{t} - 20 = 12$ (iii) -32

Question 10

- (a) The response to this question was revealing in that only a small minority of candidates understood what was meant by a histogram. The majority of candidates attempting this question ignored this, and instead of having Frequency Density on the vertical axis, had Frequency only. Most candidates used the suggested scale on the horizontal axis. A few frequency polygons were seen.
- (b) Again, what was required here was not appreciated by many candidates. Answers included several of the intervals indicated in the table, as well as individual numbers such as 95.
- (c) Candidates were more comfortably prepared for this part of the question. Some accurate work was seen with careful and consistent use of mid-interval values. The use of the interval width in the calculation of $\sum fx$ is still a common error. Division of this by 130 was also common.
- (d) Candidates were reasonably successful with this probability.
- (e) (i) The probability of this combined event requires the product of two probabilities evaluated from the table. Only a minority of candidates managed this successfully. Correct starts were sometimes seen with $\frac{32}{80}$ but this was not always followed by $\frac{31}{79}$. Sometimes the correct product was multiplied by 2.
- (ii) Again there was very limited success in this question. The combined event in this case can be arrived at in 2 ways, so we could have either $\frac{4}{80} \times \frac{8}{79} + \frac{8}{80} \times \frac{4}{79}$ or $2 \times \frac{4}{80} \times \frac{8}{79}$. This time, the multiplication by 2 was sometimes omitted. Candidates are often unsure when to add and when to multiply the probabilities of the related single events. In the work of weaker candidates, it is still fairly common to see the use of probabilities greater than 1.

Answers: (a) Correct histogram (b) $95 < t \leq 100$ (c) 98.2 (d) $\frac{28}{80}$ (e)(i) $\frac{992}{6320}$ (ii) $\frac{64}{6320}$

Question 11

- (a) (i) Where the formula was known, it was generally evaluated correctly.
- (ii) There were many good solutions, often with $\overrightarrow{AF} = \overrightarrow{AH} + \overrightarrow{HF}$ clearly stated.
- (iii)(a) Again, this was often well done.
- (b) Candidates choosing this question who had correctly answered the previous part were usually able to give a clear explanation here.
- (iv) The relationship between the vectors and the coordinates was not so well understood.
- (b) (i) The flagstaff was sometimes omitted from the translated shape. Otherwise candidates seemed to understand what was required here.
- (ii) Full descriptions, requiring the scale factor and centre, were seen. Quite a number of candidates gave only one of these. Despite the wording of the question, there were occasional mentions of other transformations.
- (iii) It was clear from the diagram that candidates knew how to work this out. There was a good proportion of correct answers.

- (iv) Correct rotations were seen together with many reasonable attempts at this more difficult problem. There were a number of flags seen that had been rotated about the point (0, 4).

Answers: (a)(i) 6.08 (ii) $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ (iii)(a) $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$ (b) $\overrightarrow{GD} = 2\overrightarrow{FH}$ (iv) (9.5, 3) (b)(i) Correct image (ii) Centre (4, 0)

Scale factor 2 (iii) (5, 2) (iv) Correct image